

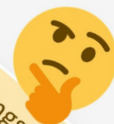
AP[®] Calculus AB 2002 Scoring Guidelines Form B

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program[®]. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

These materials were produced by Educational Testing Service[®] (ETS[®]), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 4,200 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT[®], the PSAT/NMSQT[®], and the Advanced Placement Program[®] (AP[®]). The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2002 by College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, SAT, and the acorn logo are registered trademarks of the College Entrance Examination Board. APIEL is a trademark owned by the College Entrance Examination Board. PSAT/NMSQT is a registered trademark jointly owned by the College Entrance Examination Board and the National Merit Scholarship Corporation. Educational Testing Service and ETS are registered trademarks of Educational Testing Service.



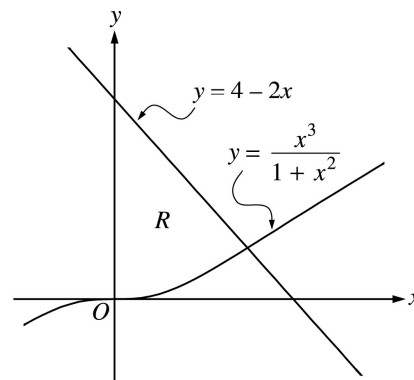
AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 1

Let R be the region bounded by the y -axis and the graphs of

$y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the x -axis.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.



Region R

$$\frac{x^3}{1+x^2} = 4 - 2x \text{ at } x = 1.487664 = A$$

$$\begin{aligned} \text{(a) Area} &= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2} \right) dx \\ &= 3.214 \text{ or } 3.215 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^A \left((4 - 2x)^2 - \left(\frac{x^3}{1+x^2} \right)^2 \right) dx \\ &= 31.884 \text{ or } 31.885 \text{ or } 10.149\pi \end{aligned}$$

$$\begin{aligned} \text{(c) Volume} &= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2} \right)^2 dx \\ &= 8.997 \end{aligned}$$

1 : Correct limits in an integral in (a), (b), or (c).

2 { 1 : integrand
1 : answer

3 { 2 : integrand and constant
< -1 > each error
1 : answer

3 { 2 : integrand
< -1 > each error
note: 0/2 if not of the form
 $k \int_c^d (f(x) - g(x))^2 dx$
1 : answer

**AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)**

Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?
- (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
- (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$
so the amount is not increasing at this time.

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$
 $t = (5 \ln 3)^2 = 30.174$
 $P'(t)$ is negative for $0 < t < (5 \ln 3)^2$ and positive for $t > (5 \ln 3)^2$. Therefore there is a minimum at $t = (5 \ln 3)^2$.

(c) $P(30.174) = 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}}) dt$
 $= 35.104 < 40$, so the lake is safe.

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

1 : answer with reason

3 $\left\{ \begin{array}{l} 1 : \text{sets } P'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{array} \right.$

3 $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{conclusion with reason} \\ \quad \text{based on integral of } P'(t) \end{array} \right.$

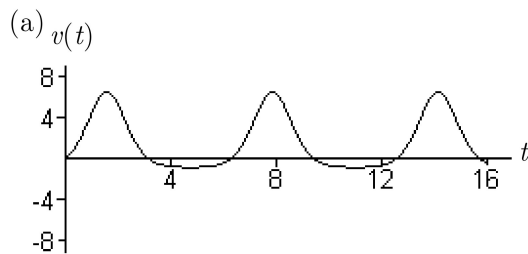
2 $\left\{ \begin{array}{l} 1 : \text{slope of tangent line} \\ 1 : \text{answer} \end{array} \right.$

**AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)**

Question 3

A particle moves along the x -axis so that its velocity v at any time t , for $0 \leq t \leq 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

- (a) On the axes provided, sketch the graph of $v(t)$ for $0 \leq t \leq 16$.
- (b) During what intervals of time is the particle moving to the left? Give a reason for your answer.
- (c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.
- (d) Is there any time t , $0 < t \leq 16$, at which the particle returns to the origin? Justify your answer.



1 : graph
 three “humps”
 periodic behavior
 starts at origin
 reasonable relative max and min values

- (b) Particle is moving to the left when $v(t) < 0$, i.e. $e^{2\sin t} < 1$.
 $(\pi, 2\pi)$, $(3\pi, 4\pi)$ and $(5\pi, 16]$

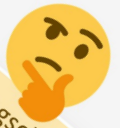
3 { 2 : intervals
 $< -1 >$ each missing or incorrect interval
 1 : reason

(c) $\int_0^4 |v(t)| dt = 10.542$
 or
 $v(t) = e^{2\sin t} - 1 = 0$
 $t = 0$ or $t = \pi$
 $x(\pi) = \int_0^\pi v(t) dt = 10.10656$
 $x(4) = \int_0^4 v(t) dt = 9.67066$
 $|x(\pi) - x(0)| + |x(4) - x(\pi)|$
 $= 10.542$

3 { 1 : limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$
 or
 uses $x(0)$ and $x(4)$ to compute distance
 1 : handles change of direction at student’s turning point
 1 : answer
 note: 0/1 if incorrect turning point

- (d) There is no such time because $\int_0^T v(t) dt > 0$ for all $T > 0$.

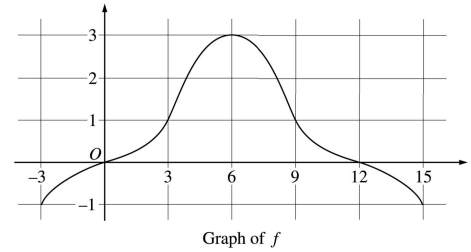
2 { 1 : no such time
 1 : reason



**AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)**

Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let



$$g(x) = 5 + \int_6^x f(t) dt \text{ for } -3 \leq x \leq 15.$$

- (a) Find $g(6)$, $g'(6)$, and $g''(6)$.
 (b) On what intervals is g decreasing? Justify your answer.
 (c) On what intervals is the graph of g concave down? Justify your answer.
 (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t) dt$ using six subintervals of length $\Delta t = 3$.

(a) $g(6) = 5 + \int_6^6 f(t) dt = 5$
 $g'(6) = f(6) = 3$
 $g''(6) = f'(6) = 0$

$$3 \begin{cases} 1 : g(6) \\ 1 : g'(6) \\ 1 : g''(6) \end{cases}$$

(b) g is decreasing on $[-3,0]$ and $[12,15]$ since
 $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.

$$3 \begin{cases} 1 : [-3,0] \\ 1 : [12,15] \\ 1 : \text{justification} \end{cases}$$

(c) The graph of g is concave down on $(6,15)$ since
 $g' = f$ is decreasing on this interval.

$$2 \begin{cases} 1 : \text{interval} \\ 1 : \text{justification} \end{cases}$$

(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$
 $= 12$

1 : trapezoidal method

**AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)**

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

- (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f . Find the x -coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.
- (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

(a) $\frac{dy}{dx} = 0$ when $x = 3$
 $\frac{d^2y}{dx^2} \Big|_{(3,-2)} = \frac{-y - y'(3-x)}{y^2} \Big|_{(3,-2)} = \frac{1}{2},$

so f has a local minimum at this point.

or

Because f is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which $y < 0$. On this interval, $\frac{dy}{dx}$ is negative to the left of $x = 3$ and $\frac{dy}{dx}$ is positive to the right of $x = 3$. Therefore f has a local minimum at $x = 3$.

(b) $y \, dy = (3-x) \, dx$
 $\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$

$8 = 18 - 18 + C ; C = 8$

$y^2 = 6x - x^2 + 16$

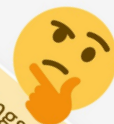
$y = -\sqrt{6x - x^2 + 16}$

$$3 \left\{ \begin{array}{l} 1 : x = 3 \\ 1 : \text{local minimum} \\ 1 : \text{justification} \end{array} \right.$$

$$6 \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } g(6) = -4 \\ 1 : \text{solves for } y \end{array} \right.$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

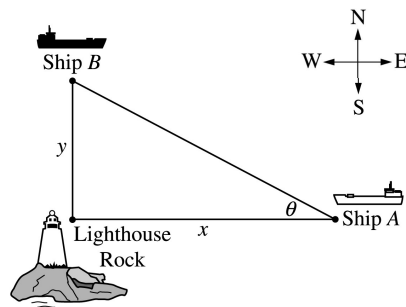
Note: 0/6 if no separation of variables



AP[®] CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 6

Ship *A* is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship *B* is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship *A* and Lighthouse Rock at time t , and let y be the distance between Ship *B* and Lighthouse Rock at time t , as shown in the figure above.



- (a) Find the distance, in kilometers, between Ship *A* and Ship *B* when $x = 4$ km and $y = 3$ km.
- (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.
- (c) Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 4$ km and $y = 3$ km.

(a) Distance = $\sqrt{3^2 + 4^2} = 5$ km

(b) $r^2 = x^2 + y^2$

$$2r \frac{dr}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

or explicitly:

$$r = \sqrt{x^2 + y^2}$$

$$\frac{dr}{dt} = \frac{1}{2\sqrt{x^2 + y^2}} \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right)$$

At $x = 4$, $y = 3$,

$$\frac{dr}{dt} = \frac{4(-15) + 3(10)}{5} = -6 \text{ km/hr}$$

(c) $\tan \theta = \frac{y}{x}$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{\frac{dy}{dt}x - \frac{dx}{dt}y}{x^2}$$

At $x = 4$ and $y = 3$, $\sec \theta = \frac{5}{4}$

$$\frac{d\theta}{dt} = \frac{16}{25} \left(\frac{10(4) - (-15)(3)}{16} \right)$$

$$= \frac{85}{25} = \frac{17}{5} \text{ radians/hr}$$

1 : answer

- 4 {
- 1 : expression for distance
 - 2 : differentiation with respect to t
 - < -2 > chain rule error
 - 1 : evaluation

- 4 {
- 1 : expression for θ in terms of x and y
 - 2 : differentiation with respect to t
 - < -2 > chain rule, quotient rule, or transcendental function error
 - note: 0/2 if no trig or inverse trig function
 - 1 : evaluation