



## AP<sup>®</sup> Calculus AB 2007 Scoring Guidelines

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**Question 1**

Let  $R$  be the region in the first and second quadrants bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line  $y = 2$ .

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.
- (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

(a) Area =  $\int_{-3}^3 \left( \frac{20}{1+x^2} - 2 \right) dx = 37.961$  or  $37.962$

(b) Volume =  $\pi \int_{-3}^3 \left( \left( \frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$

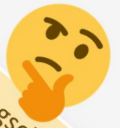
(c) Volume =  $\frac{\pi}{2} \int_{-3}^3 \left( \frac{1}{2} \left( \frac{20}{1+x^2} - 2 \right) \right)^2 dx$   
 $= \frac{\pi}{8} \int_{-3}^3 \left( \frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in (a), (b), or (c)

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$



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### Question 2

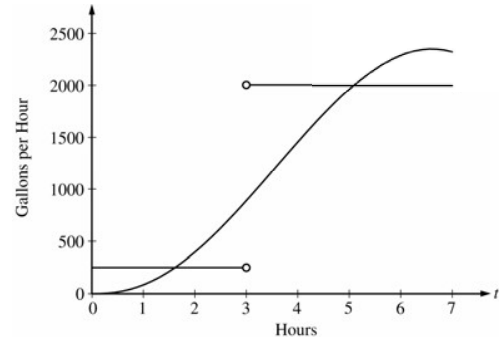
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.
- For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a)  $\int_0^7 f(t) dt \approx 8264$  gallons

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals  $0 \leq t \leq 1.617$  and  $3 \leq t \leq 5.076$  because  $f(t) < g(t)$  for  $0 \leq t < 1.617$  and  $3 < t < 5.076$ .

2 :  $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since  $f(t) - g(t)$  changes sign from positive to negative only at  $t = 3$ , the candidates for the absolute maximum are at  $t = 0, 3$ , and  $7$ .

5 :  $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

$t$ (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

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**Question 3**

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions  $f$  and  $g$  are differentiable for all real numbers, and  $g$  is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of  $x$ . The function  $h$  is given by  $h(x) = f(g(x)) - 6$ .

- (a) Explain why there must be a value  $r$  for  $1 < r < 3$  such that  $h(r) = -5$ .
- (b) Explain why there must be a value  $c$  for  $1 < c < 3$  such that  $h'(c) = -5$ .
- (c) Let  $w$  be the function given by  $w(x) = \int_1^{g(x)} f(t) dt$ . Find the value of  $w'(3)$ .
- (d) If  $g^{-1}$  is the inverse function of  $g$ , write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at  $x = 2$ .

(a)  $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$   
 $h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$   
 Since  $h(3) < -5 < h(1)$  and  $h$  is continuous, by the Intermediate Value Theorem, there exists a value  $r$ ,  $1 < r < 3$ , such that  $h(r) = -5$ .

2 :  $\begin{cases} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{cases}$

(b)  $\frac{h(3) - h(1)}{3 - 1} = \frac{-7 - 3}{3 - 1} = -5$   
 Since  $h$  is continuous and differentiable, by the Mean Value Theorem, there exists a value  $c$ ,  $1 < c < 3$ , such that  $h'(c) = -5$ .

2 :  $\begin{cases} 1 : \frac{h(3) - h(1)}{3 - 1} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c)  $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

2 :  $\begin{cases} 1 : \text{apply chain rule} \\ 1 : \text{answer} \end{cases}$

(d)  $g(1) = 2$ , so  $g^{-1}(2) = 1$ .  
 $(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$

3 :  $\begin{cases} 1 : g^{-1}(2) \\ 1 : (g^{-1})'(2) \\ 1 : \text{tangent line equation} \end{cases}$

An equation of the tangent line is  $y - 1 = \frac{1}{5}(x - 2)$ .

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**Question 4**

A particle moves along the  $x$ -axis with position at time  $t$  given by  $x(t) = e^{-t} \sin t$  for  $0 \leq t \leq 2\pi$ .

- (a) Find the time  $t$  at which the particle is farthest to the left. Justify your answer.  
 (b) Find the value of the constant  $A$  for which  $x(t)$  satisfies the equation  $Ax''(t) + x'(t) + x(t) = 0$  for  $0 < t < 2\pi$ .

(a)  $x'(t) = -e^{-t} \sin t + e^{-t} \cos t = e^{-t} (\cos t - \sin t)$   
 $x'(t) = 0$  when  $\cos t = \sin t$ . Therefore,  $x'(t) = 0$  on  
 $0 \leq t \leq 2\pi$  for  $t = \frac{\pi}{4}$  and  $t = \frac{5\pi}{4}$ .  
 The candidates for the absolute minimum are at  
 $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$ , and  $2\pi$ .

$t$	$x(t)$
0	$e^0 \sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}} \sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}} \sin\left(\frac{5\pi}{4}\right) < 0$
$2\pi$	$e^{-2\pi} \sin(2\pi) = 0$

The particle is farthest to the left when  $t = \frac{5\pi}{4}$ .

(b)  $x''(t) = -e^{-t} (\cos t - \sin t) + e^{-t} (-\sin t - \cos t)$   
 $= -2e^{-t} \cos t$   
 $Ax''(t) + x'(t) + x(t)$   
 $= A(-2e^{-t} \cos t) + e^{-t} (\cos t - \sin t) + e^{-t} \sin t$   
 $= (-2A + 1)e^{-t} \cos t$   
 $= 0$   
 Therefore,  $A = \frac{1}{2}$ .

5 :  $\begin{cases} 2 : x'(t) \\ 1 : \text{sets } x'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

4 :  $\begin{cases} 2 : x''(t) \\ 1 : \text{substitutes } x''(t), x'(t), \text{ and } x(t) \\ \text{into } Ax''(t) + x'(t) + x(t) \\ 1 : \text{answer} \end{cases}$

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### Question 5

$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ . (Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)

- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.

(a)  $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$  ft  
 Since the graph of  $r$  is concave down on the interval  $5 < t < 5.4$ , this estimate is greater than  $r(5.4)$ .

(b)  $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$   
 $\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2(2) = 7200\pi$  ft<sup>3</sup>/min

(c)  $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$   
 $= 19.3$  ft  
 $\int_0^{12} r'(t) dt$  is the change in the radius, in feet, from  $t = 0$  to  $t = 12$  minutes.

(d) Since  $r$  is concave down,  $r'$  is decreasing on  $0 < t < 12$ . Therefore, this approximation, 19.3 ft, is less than  $\int_0^{12} r'(t) dt$ .

Units of ft<sup>3</sup>/min in part (b) and ft in part (c)

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

3 :  $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

1 : conclusion with reason

1 : units in (b) and (c)

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**Question 6**

Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant.

- (a) Find  $f'(x)$  and  $f''(x)$ .
- (b) For what value of the constant  $k$  does  $f$  have a critical point at  $x = 1$ ? For this value of  $k$ , determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 1$ . Justify your answer.
- (c) For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .

(a)  $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$

$$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$$

$$2 : \begin{cases} 1 : f'(x) \\ 1 : f''(x) \end{cases}$$

(b)  $f'(1) = \frac{1}{2}k - 1 = 0 \Rightarrow k = 2$

When  $k = 2$ ,  $f'(1) = 0$  and  $f''(1) = -\frac{1}{2} + 1 > 0$ .  
 $f$  has a relative minimum value at  $x = 1$  by the Second Derivative Test.

$$4 : \begin{cases} 1 : \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1 : \text{solves for } k \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(c) At this inflection point,  $f''(x) = 0$  and  $f(x) = 0$ .

$$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$$

$$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$$

$$3 : \begin{cases} 1 : f''(x) = 0 \text{ or } f(x) = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{answer} \end{cases}$$

Therefore,  $\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$   
 $\Rightarrow 4 = \ln x$   
 $\Rightarrow x = e^4$   
 $\Rightarrow k = \frac{4}{e^2}$